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ESTIMATION OF THE PARAMETERS OF THE GOMPERTZ DISTRIBUTION UNDER THE FIRST FAILURE-CENSORED SAMPLING PLAN

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In this paper, we provide a method for constructing an exact confidence interval and an exact joint confidence region for the parameters of the Gompertz distribution under the first failure-censored sampling plan [1]. Moreover, when compared to ordinary sampling plans, the sampling plan of Balasooriya [1] has an advantage in terms of shortening test-time and a saving of resources. Finally, we give an example to illustrate our proposed method. Results from simulation studies assessing the performance of our proposed method are included.

Keywords: First failure-censored; Gompertz distribution; Joint confidence region

1991 AMS Classification: 62F25, 62G30

1 INTRODUCTION

The Gompertz distribution occupies an important position in modelling human mortality and fitting actuarial tables. Historically, the Gompertz distribution was introduced by Gompertz [5]. In recent years, many authors have contributed to the statistical methodology and characterization of this distribution; for example, Read [9], Gordon [6], Makany [7], Rao and Damaraju [8], Franses [3] and Wu and Lee [10]. Garg *et al*. [4] studied the properties of the Gompertz distribution and obtained the maximum likelihood estimates for the parameters. Chen [2] developed an exact confidence interval and an exact joint confidence region for the parameters of the Gompertz distribution under type II censoring.

The first failure-censored sampling plan, which was developed by Balasooriya [1], consists of grouping a number of specimens into several sets or assemblies of the same size and testing each of these assemblies of specimens separately until the occurrence of first failure in each assembly. Furthermore, he examined this sampling plan for the two-parameters exponential distribution based on m random samples (or assemblies) of equal size n. He also showed that this sampling plan has an advantage in terms of shortening test-time and saving of resources over an ordinary sampling plan of testing mn units.

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Therefore, the purpose of this paper is to construct an exact confidence interval and an exact joint confidence region for the parameters of the Gompertz distribution under the first failure-censored sampling plan. Finally, we will give an example to illustrate our proposed method. Results from simulation studies assessing the performance of our proposed method are included.

2 MAIN RESULTS

Let X be the lifetime of a product with the probability density function (p.d.f.) as given by

$$f(x; c, \lambda) = \lambda e^{cx} \exp\left\{-\frac{\lambda}{c}(e^{cx} - 1)\right\}, \quad x > 0 \tag{1}$$

where $c > 0$ and $\lambda > 0$ are the parameters, respectively. It is worth noting that when $c \to 0$, the Gompertz distribution will tend to an exponential distribution. Let $X_{(1)} < X_{(2)} < \cdots < X_{(n)}$ be the order statistics of a random sample of size n from (1). The p.d.f. of the first-order statistics $X_{(1)}$ is

$$f(x; c, \lambda^*) = \lambda^* e^{cx} \exp\left\{-\frac{\lambda^*}{c}(e^{cx} - 1)\right\}, \tag{2}$$

where $\lambda^* = n\lambda$.

Let $\{X_{(1)1}, X_{(1)2}, \ldots, X_{(1)m}\}$ denote the set of first-order statistics of m samples of size n from (1) and let $X'_{(1)} < X'_{(2)} < \cdots < X'_{(m)}$ be the corresponding order statistics. Clearly, $X_{(1)1}, X_{(1)2}, \ldots, X_{(1)m}$ can also be considered as a random sample from (2). Define

$$Y_i = \frac{\lambda^*}{c}(e^{cX_{(1)i}} - 1), \quad i = 1, \ldots, m.$$

Then $Y_1, Y_2, \ldots, Y_m$ form a sample from the standard exponential distribution. Let $Y_{(1)} < Y_{(2)} < \cdots < Y_{(m)}$ be the corresponding order statistics. Since the function

$$g(x) = \frac{\lambda^*}{c}(e^{cx} - 1)$$

is strictly increasing in x, then

$$Y_{(i)} = \frac{\lambda^*}{c}(e^{cX'_{(i)}} - 1), \quad i = 1, \ldots, m.$$

Let

$$U = 2\sum_{i=1}^{m}(Y_{(i)} - Y_{(1)}) \quad \text{and} \quad V = 2mY_{(1)}.$$

Then U and V are independent random variables. Both U and V have χ^2 distribution with $2m - 2$ and 2 degrees of freedom, respectively. Define

$$\xi = \frac{U}{(m-1)V} = \frac{\sum_{i=1}^{m}\left(e^{cX'_{(i)}} - e^{cX'_{(1)}}\right)}{m(m-1)(e^{cX'_{(1)}} - 1)}$$

and

$$\zeta = U + V = \frac{2n\lambda}{c}\sum_{i=1}^{m}(e^{cX'_{(i)}} - 1).$$

To derive an exact confidence interval for the parameter c and an exact joint confidence region for the parameters λ and c, the following lemmas are necessary. Lemma 1 is based on the above discussion.

LEMMA 1 *Let ξ and ζ be defined as above. Then ξ has an F distribution with $2m-2$ and 2 degrees of freedom and ζ has a χ^2 distribution with $2m$ degrees of freedom. Furthermore, these two random variables are independent.*

LEMMA 2 *Suppose that $0 < a_1 < a_2 < \cdots < a_m$ and $t > 0$. If*

$$t \neq \frac{\sum_{i=1}^{m}(a_i - a_1)}{m(m-1)a_1},$$

then the equation

$$\frac{\sum_{i=1}^{m}(e^{ca_i} - e^{ca_1})}{m(m-1)(e^{ca_1} - 1)} = t$$

has a unique solution for $c \neq 0$.

Proof Note that the function

$$h(c) = \frac{\sum_{i=1}^{m}(e^{ca_i} - e^{ca_1})}{m(m-1)(e^{ca_1} - 1)}$$

is strictly increasing in c for any $c \neq 0$, that

$$\lim_{c\to 0-} h(c) = \lim_{c\to 0+} h(c) = \frac{\sum_{i=1}^{m}(a_i - a_1)}{m(m-1)a_1}, \quad \lim_{c\to\infty} h(c) = \infty, \quad \lim_{c\to -\infty} h(c) = 0.$$

Then the proof follows. ■

The next two theorems provide an exact confidence interval for the parameter c and an exact joint confidence region for the parameters λ and c of the Gompertz distribution under the first failure-censored sampling plan [1]. In the following discussion, let $F_\alpha(\nu_1, \nu_2)$ be the upper α critical value of the F distribution with ν_1 and ν_2 degrees of freedom and let $\chi^2_\alpha(\nu)$ be the upper α critical value of the χ^2 distribution with ν degrees of freedom.

THEOREM 1 *Let $X'_{(1)}, X'_{(2)}, \ldots, X'_{(m)}$ be defined as above. Then for any $0 < \alpha < 1$,*

$$\phi(X'_{(1)}, \ldots, X'_{(m)}, F_{1-\alpha/2}(2m-2, 2)) < c < \phi(X'_{(1)}, \ldots, X'_{(m)}, F_{\alpha/2}(2m-2, 2)) \qquad (3)$$

is a $100(1-\alpha)\%$ confidence interval for the parameter c, where

$$\phi(X'_{(1)}, \ldots, X'_{(m)}, t)$$

is the solution of c for the equation

$$\frac{\sum_{i=1}^{m}(e^{cx'_{(i)}} - e^{cx'_{(1)}})}{m(m-1)(e^{cx'_{(1)}} - 1)} = t.$$

Proof By Lemmas 1 and 2, we have

$$\begin{aligned}
&P\{\phi(X'_{(1)}, \ldots, X'_{(m)}, F_{1-\alpha/2}(2m-2,2)) < c < \phi(X'_{(1)}, \ldots, X'_{(m)}, F_{\alpha/2}(2m-2,2))\} \\
&\quad = P\left\{F_{1-\alpha/2}(2m-2,2) < \frac{\sum_{i=1}^{m}(e^{cX'_{(i)}} - e^{cX'_{(1)}})}{m(m-1)(e^{cX'_{(1)}} - 1)} < F_{\alpha/2}(2m-2,2)\right\} \\
&\quad = 1-\alpha.
\end{aligned}$$

The proof is completed. ■

If $F_{1-\alpha/2}(2m-2,2) < [\sum_{i=1}^{m}(x'_{(i)} - x'_{(1)})/(m(m-1)x'_{(1)})]$, then the lower confidence limit for c obtained by (3) is negative. Hence, we should use 0 as the lower limit. The other option is to find a $100(1-\alpha)\%$ upper confidence limit c_u for c. Then $(0, c_u)$ is a $100(1-\alpha)\%$ one-sided confidence interval for c.

COROLLARY 1 *Let $X'_{(1)}, X'_{(2)}, \ldots, X'_{(m)}$ be defined as above. Then for any $0 < \alpha < 1$,*

$$\phi(X'_{(1)}, \ldots, X'_{(m)}, F_{\alpha}(2m-2,2))$$

is a $100(1-\alpha)\%$ upper confidence limit for the parameter c, where

$$\phi(X'_{(1)}, \ldots, X'_{(m)}, t)$$

is defined in Theorem 1.

THEOREM 2 *Let $X'_{(1)}, X'_{(2)}, \ldots, X'_{(m)}$ be defined as above. Then for any $0 < \alpha < 1$, the following inequalities determine a $100(1-\alpha)\%$ joint confidence region for λ and c:*

$$\begin{cases}
\phi(X'_{(1)}, \ldots, X'_{(m)}, F_{(1+\sqrt{1-\alpha})/2}(2m-2,2)) < c \\
\qquad < \phi(X'_{(1)}, \ldots, X'_{(m)}, F_{(1-\sqrt{1-\alpha})/2}(2m-2,2)) \\
\dfrac{c\chi^2_{(1+\sqrt{1-\alpha})/2}(2m)}{2n\sum_{i=1}^{m}(e^{cX'_{(i)}} - 1)} < \lambda < \dfrac{c\chi^2_{(1-\sqrt{1-\alpha})/2}(2m)}{2n\sum_{i=1}^{m}(e^{cX'_{(i)}} - 1)}
\end{cases}$$

where

$$\phi(X'_{(1)}, \ldots, X'_{(m)}, t)$$

is defined in Theorem 1.

Proof By Lemmas 1 and 2, we have

$$
\begin{aligned}
P\Bigg\{&\phi(X'_{(1)}, \ldots, X'_{(m)}, F_{(1+\sqrt{1-\alpha})/2}(2m-2,2)) < c \\
&< \phi(X'_{(1)}, \ldots, X'_{(m)}, F_{(1-\sqrt{1-\alpha})/2}(2m-2,2)), \\
&\frac{c\chi^2_{(1+\sqrt{1-\alpha})/2}(2m)}{2n\sum_{i=1}^m (e^{cX'_{(i)}}-1)} < \lambda < \frac{c\chi^2_{(1-\sqrt{1-\alpha})/2}(2m)}{2n\sum_{i=1}^m (e^{cX'_{(i)}}-1)}\Bigg\} \\
&= P\left\{F_{(1+\sqrt{1-\alpha})/2}(2m-2,2) < \frac{\sum_{i=1}^m (e^{cX'_{(i)}} - e^{cX'_{(1)}})}{m(m-1)(e^{cX'_{(1)}}-1)} < F_{(1-\sqrt{1-\alpha})/2}(2m-2,2)\right\} \\
&\quad \times P\left\{\chi^2_{(1+\sqrt{1-\alpha})/2}(2m) < \frac{2n\lambda}{c}\sum_{i=1}^m (e^{cX'_{(i)}}-1) < \chi^2_{(1-\sqrt{1-\alpha})/2}(2m)\right\} \\
&= 1-\alpha.
\end{aligned}
$$

The proof is completed. ■

Similar to Corollary 1, another form of the joint confidence region of the parameters c and λ can be obtained as follows.

COROLLARY 2 *Let* $X'_{(1)}, X'_{(2)}, \ldots, X'_{(m)}$ *be defined as above. Then for any* $0 < \alpha < 1$, *the following inequalities determine a* $100(1-\alpha)\%$ *joint confidence region for* λ *and* c:

$$
\begin{cases}
0 < c < \phi(X'_{(1)}, \ldots, X'_{(m)}, F_{1-\sqrt{1-\alpha}}(2m-2,2)) \\
\dfrac{c\chi^2_{(1+\sqrt{1-\alpha})/2}(2m)}{2n\sum_{i=1}^m (e^{cX'_{(i)}}-1)} < \lambda < \dfrac{c\chi^2_{(1-\sqrt{1-\alpha})/2}(2m)}{2n\sum_{i=1}^m (e^{cX'_{(i)}}-1)}
\end{cases}
$$

where

$$\phi(X'_{(1)}, \ldots, X'_{(m)}, t)$$

is defined in Theorem 1.

3 AN EXAMPLE

To illustrate the use of our proposed method, we have a simulated data consisting of 50 observations from a Gompertz distribution with $c = 0.05$, $\lambda = 0.01$ are randomly grouped into 5 sets. The ordered observations in each set are shown in Table I. Based on Table I, we know that

$$x'_{(1)} = 25.30, \quad x'_{(2)} = 26.45, \quad x'_{(3)} = 29.79, \quad x'_{(4)} = 42.24, \quad x'_{(5)} = 49.02.$$

TABLE I Ordered Observations in Each Set.

Set 1	*Set 2*	*Set 3*	*Set 4*	*Set 5*
42.24	29.79	26.45	49.02	25.30
45.37	36.26	28.37	49.99	29.41
49.52	39.26	29.99	50.74	32.32
54.62	40.21	43.33	59.95	38.49
56.67	46.87	45.65	68.00	39.22
61.09	52.02	53.32	75.23	46.64
67.91	65.98	58.87	76.35	46.73
77.78	75.45	60.90	80.11	55.22
81.23	75.64	65.66	84.66	56.98
88.45	81.26	90.09	87.27	78.87

To construct 95% two-sided and 95% one-sided confidence intervals for the parameter c, note that

$$F_{0.975}(8,2) = 0.165, \quad F_{0.025}(8,2) = 39.4 \quad \text{and} \quad F_{0.05}(8,2) = 19.37.$$

It can be found that 0.02688 is the solution of c for the equation

$$\frac{\sum_{i=1}^{5}(e^{cx'_{(i)}} - e^{cx'_{(1)}})}{20(e^{cx'_{(1)}} - 1)} = 0.165,$$

and that 0.27501 is the solution of c for the equation

$$\frac{\sum_{i=1}^{5}(e^{cx'_{(i)}} - e^{cx'_{(1)}})}{20(e^{cx'_{(1)}} - 1)} = 39.4,$$

and that 0.24373 is the solution of c for the equation

$$\frac{\sum_{i=1}^{5}(e^{cx'_{(i)}} - e^{cx'_{(1)}})}{20(e^{cx'_{(1)}} - 1)} = 19.37.$$

Thus (0.02688, 0.27501) and (0, 0.24373) are 95% two-sided and 95% one-sided confidence intervals for the parameter c by Theorem 1 and Corollary 1, respectively. Furthermore,

$$\chi^2_{0.975}(10) = 3.247 \quad \text{and} \quad \chi^2_{0.025}(10) = 20.48.$$

Then a 90% joint confidence region for the parameters λ and c is determined by the following inequalities:

$$\begin{cases} 0.02688 < c < 0.27501 \\ \dfrac{1.6235c}{10\left(\sum_{i=1}^{5} e^{cx'_{(i)}} - 5\right)} < \lambda < \dfrac{10.24c}{10\left(\sum_{i=1}^{5} e^{cx'_{(i)}} - 5\right)} \end{cases}$$

by Theorem 2. Alternatively, the 90% joint confidence region can be determined by the following:

$$\begin{cases} 0 < c < 0.24373 \\ \dfrac{1.6235c}{10\left(\sum_{i=1}^{5} e^{cx'_{(i)}} - 5\right)} < \lambda < \dfrac{10.24c}{10\left(\sum_{i=1}^{5} e^{cx'_{(i)}} - 5\right)}. \end{cases}$$

4 SIMULATION STUDY

In this section, we will report the results of a simulation study for constructing a confidence interval and confidence region for the parameter c and the parameters c and λ by Theorems 1 and 2. We considered α values of 0.01 and 0.05 and the generated sample from Gompertz distribution with $c = 0.01, 0.1$ and $\lambda = 0.01, 0.02$. For each case, we estimate a significance level via 100 confidence intervals for c and 100 confidence regions for c and λ. Based on 1000 empirical significance levels, say $\hat{\alpha}_1, \ldots, \hat{\alpha}_{1000}$, we compute

$$\hat{\alpha} = \frac{1}{1000}\sum_{i=1}^{1000}\hat{\alpha}_i \quad \text{and} \quad \hat{se} = \left\{\frac{1}{1000}\sum_{i=1}^{1000}(\hat{\alpha}_i - \alpha)^2\right\}^{1/2}.$$

TABLE II Average Empirical Significance Level Based on $100(1-\alpha)\%$ Confidence Interval for c When $\alpha = 0.05$.

	m		
n	5	10	30
	$(c, \lambda) = (0.01, 0.01)$		
10	0.0644 (0.0038)	0.0484 (0.0024)	0.0418 (0.0018)
30	0.0501 (0.0025)	0.0482 (0.0020)	0.0410 (0.0015)
	$(c, \lambda) = (0.01, 0.02)$		
10	0.0687 (0.0035)	0.0485 (0.0028)	0.0435 (0.0025)
30	0.0654 (0.0032)	0.0481 (0.0026)	0.0411 (0.0022)
	$(c, \lambda) = (0.1, 0.01)$		
10	0.0624 (0.0040)	0.0481 (0.0030)	0.0425 (0.0021)
30	0.0612 (0.0035)	0.0439 (0.0029)	0.0432 (0.0019)
	$(c, \lambda) = (0.1, 0.02)$		
10	0.0587 (0.0039)	0.0481 (0.0037)	0.0424 (0.0028)
30	0.0502 (0.0036)	0.0457 (0.0033)	0.0416 (0.0025)

Note: m denotes the number of assemblies or sets, and n is the sample size in each assembly. The values in parentheses are sample mean squared error of $\hat{\alpha}$.

TABLE III Average Empirical Significance Level Based on $100(1-\alpha)\%$ Confidence Interval for c When $\alpha = 0.01$.

	m		
n	5	10	30
	$(c, \lambda) = (0.01, 0.01)$		
10	0.0146 (0.0021)	0.0096 (0.0018)	0.0092 (0.0016)
30	0.0118 (0.0019)	0.0096 (0.0014)	0.0098 (0.0012)
	$(c, \lambda) = (0.01, 0.02)$		
10	0.0131 (0.0021)	0.0097 (0.0019)	0.0097 (0.0014)
30	0.0107 (0.0015)	0.0096 (0.0012)	0.0096 (0.0011)
	$(c, \lambda) = (0.1, 0.01)$		
10	0.0115 (0.0025)	0.0098 (0.0022)	0.0097 (0.0016)
30	0.0108 (0.0021)	0.0096 (0.0017)	0.0091 (0.0013)
	$(c, \lambda) = (0.1, 0.02)$		
10	0.0115 (0.0024)	0.0097 (0.0018)	0.0095 (0.0016)
30	0.0104 (0.0022)	0.0095 (0.0017)	0.0088 (0.0015)

Note: m denotes the number of assemblies or sets, and n is the sample size in each assembly. The values in parentheses are sample mean squared error of $\hat{\alpha}$.

TABLE IV Average Empirical Significance Level Based on $100(1-\alpha)\%$ Confidence Region for c When $\alpha = 0.05$.

n	m = 5	m = 10	m = 30
	$(c, \lambda) = (0.01, 0.01)$		
10	0.0535 (0.0029)	0.0494 (0.0026)	0.0446 (0.0021)
30	0.0513 (0.0023)	0.0482 (0.0022)	0.0426 (0.0019)
	$(c, \lambda) = (0.01, 0.02)$		
10	0.0569 (0.0030)	0.0497 (0.0027)	0.0456 (0.0022)
30	0.0583 (0.0026)	0.0489 (0.0024)	0.0427 (0.0020)
	$(c, \lambda) = (0.1, 0.01)$		
10	0.0543 (0.0034)	0.0476 (0.0030)	0.0455 (0.0027)
30	0.0546 (0.0031)	0.0457 (0.0026)	0.0438 (0.0023)
	$(c, \lambda) = (0.1, 0.02)$		
10	0.0542 (0.0035)	0.0489 (0.0033)	0.0448 (0.0025)
30	0.0520 (0.0032)	0.0478 (0.0026)	0.0428 (0.0021)

Note: m denotes the number of assemblies or sets, and n is the sample size in each assembly. The values in parentheses are sample mean squared error of $\hat{\alpha}$.

The results are summarized in Tables II–V. In Tables II–V, m denotes the number of assemblies or sets, and n is the sample size in each assembly. Hence, from Tables II and III and based on $100(1-\alpha)\%$ confidence interval for c, it appears clear that (a) as n increases, average empirical significance levels approach the nominal values for $m = 5$; (b) as n decreases, average empirical significance levels approach the nominal values for $m = 10$, 30 except $(c, \lambda, m, \alpha) = (0.1, 0.01, 30, 0.05)$ and $(0.01, 0.01, 30, 0.01)$; (c) for fixed n, as m increases the average empirical significance level decreases from values higher than the nominal values to values lower than the nominal values. Furthermore, all of the average empirical significance levels close to the nominal values except $(c, \lambda, m, \alpha) = (0.01, 0.02, 5, 0.05)$. Especially, for $m = 10$, generally the performance of average empirical significance levels is better than $m = 5$ and $m = 30$, respectively. Moreover, from Tables IV and V and based on $100(1-\alpha)\%$ confidence

TABLE V Average Empirical Significance Level Based on $100(1-\alpha)\%$ Confidence Region for c When $\alpha = 0.01$.

n	m = 5	m = 10	m = 30
	$(c, \lambda) = (0.01, 0.01)$		
10	0.0106 (0.0017)	0.0098 (0.0014)	0.0096 (0.0012)
30	0.0111 (0.0016)	0.0095 (0.0013)	0.0095 (0.0010)
	$(c, \lambda) = (0.01, 0.02)$		
10	0.0110 (0.0014)	0.0101 (0.0013)	0.0098 (0.0012)
30	0.0103 (0.0013)	0.0098 (0.0011)	0.0096 (0.0010)
	$(c, \lambda) = (0.1, 0.01)$		
10	0.0113 (0.0019)	0.0102 (0.0018)	0.0095 (0.0011)
30	0.0102 (0.0018)	0.0096 (0.0015)	0.0093 (0.0012)
	$(c, \lambda) = (0.1, 0.02)$		
10	0.0121 (0.0020)	0.0099 (0.0016)	0.0096 (0.0014)
30	0.0114 (0.0019)	0.0098 (0.0013)	0.0089 (0.0009)

Note: m denotes the number of assemblies or sets, and n is the sample size in each assembly. The values in parentheses are sample mean squared error of $\hat{\alpha}$.

region for c and λ, it appears clear that (d) as n increases, average empirical significance levels approach the nominal values for $m = 5$ except $(c, \lambda, m, \alpha) = (0.01, 0.02, 5, 0.05)$, $(0.1, 0.01, 5, 0.05)$ and $(0.01, 0.01, 5, 0.01)$; (e) as n decreases, average empirical significance levels approach the nominal values for $m = 10, 30$; (f) for fixed n, as m increases the average empirical significance level decreases from values higher than the nominal values to values lower than the nominal values. Furthermore, all of the average empirical significance levels close to the nominal values. Especially, for $m = 10$, generally the performance of average empirical significance levels is better than $m = 5$ and $m = 30$, respectively.

5 CONCLUDING REMARKS

The purpose of this paper is to develop an exact confidence interval and an exact joint confidence region for the parameters of the Gompertz distribution under the first failure-censored sampling plan [1]. This sampling plan is quite useful to practitioners, because they provide savings in resources and in total test time. Results from simulation studies illustrate that the performance of our proposed method is acceptable.

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